On the Throughput of Ad-Hoc Networks using Connectivity Graphs

Christina Fragouli*
EPFL Lausanne
christina.fragouli@epfl.ch

Tarik Tabet †
EPFL Lausanne
tarik.tabet@epfl.ch

Cem Derdiyok
EPFL Lausanne
cem.derdiyok@epfl.ch

Abstract

In this paper we propose a set of necessary and sufficient conditions under which the throughput in an ad-hoc network can remain constant as the number of nodes n increases. Throughput refers to the minimum achievable rate between a source-destination pair for a given routing mechanism and physical model, when the network is shared by $\Theta(n)$[1] randomly chosen source-destination pairs.The main idea is to use a *connectivity graph*, that does not represent the actual physical network, but rather the available communication resources.

1 Introduction

In the seminal work [1], Gupta and Kumar proposed a new approach to study wireless ad-hoc networks by modeling them as random geometric graphs and examining the scaling of the capacity with the number of nodes n. Their work is important since it offers a promising way to better understand the behaviour of ad hoc networks that are difficult to analyze by standard tools. However, their results were in a way disappointing: given n nodes randomly located on a unit area disk and $\Theta(n)$ randomly selected source-destination pairs, the throughput per source-destination pair is upper-bounded to scale with n as $\Theta(1/\sqrt{n})$, leading to a vanishing throughput as the number of nodes grows.

The follow-up paper [2] by Grossglausser and Tse showed the surprising result that a constant $\Theta(1)$ throughput can be achieved per source-destination pair even if the number of nodes grows to infinity, by allowing the nodes to move. More specifically, their result assumes a uniform 2-dimensional mobility pattern, where every node is close to every other with equal probability. The next natural question to ask is whether this good performance is specific to the particular generous mobility pattern, or whether it can be achieved under more restricted conditions.

Diggavi, Grossglausser and Tse [3] made significant progress in answering this question, by demonstrating that the same order of throughput can still be achieved under

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[1]We use Knuth's notation: $f(n) = O(g(n))$ means that there exists a constant c and integer N such that $f(n) \leq cg(n)$ for $n > N$; $f(n) = \Theta(g(n))$ denotes that $f(n) = O(g(n))$ as well as $g(n) = O(f(n))$.

under what conditions on the mobility patterns of the users a throughput of $\Theta(1)$ is achievable.

The main contribution of this paper is a method that allows to easily check whether for a given network, a constant $\Theta(1)$ throughput per source-destination pair is possible.

Intuitively, the reason we get a throughput hit in [1], is that the average number of hops that a packet needs to traverse from a source to a destination scales as $\Theta(\sqrt{n})$. On the contrary, in [2, 3], mobility enables a routing strategy where the number of hops is limited to at most two. Our work was motivated by the observation that in a complete graph between each source-destination pair there exists a set of max-flow paths whose length is upper bounded by two. A graph is complete if any two nodes are connected with an edge, which intuitively seems to correspond to the fact that in a uniform mobility pattern any two nodes can become neighbors and successfully transmit to each other. We make this loose connection precise by introducing the *connectivity graph*, that does not represent the actual physical network, but rather the available communication resources.

The connectivity graph offers an abstraction of the communication capabilities of the ad-hoc network: we can study the long-term averaged throughput between source-destination pairs in the actual ad-hoc network, by examining information flows in the connectivity graph. We can actually use the properties of the connectivity graph to develop a set of necessary and sufficient conditions under which constant $\Theta(1)$ throughput is possible. We illustrate how these conditions apply to a number of different topologies, including [1, 2, 3]. We then try to understand what structural properties the necessary conditions imply for the connectivity graph and how these translate in properties for the underlying mobility pattern. Interestingly, we provide an example where constant throughput may be possible to achieve by mobile nodes that have a restricted number of neighbors. That is, each node may successfully communicate with at most $n^{1/t}$ other nodes, for a finite t.

The paper is organized as follows. Section 2 reviews the context of our work, and introduces the concept of connectivity graph. Section 3 presents the proposed set of conditions. Section 4 applies these conditions to different topologies. Section 5 investigates what structural properties of the connectivity graph imply for the underlying mobility pattern. Section 6 concludes the paper.

2 Connectivity Graph

Closely following the model in [1, 2, 3], consider a wireless ad-hoc network with n nodes that can act as transmitters and receivers. Assume discrete time, that is, time is divided into equal slots. During each time slot one information unit (packet) can be transmitted from each transmitter. Also assume a given mobility pattern. For example, in [1] nodes are immobile, while in [3] nodes move on great circles. Randomly choose $\Theta(n)$ nodes as sources. For each source S_i, randomly choose a different node D_i as destination. We distinguish between the terms "transmitter-receiver" that may refer to intermediate nodes acting as relays along an information flow path, and the terms "source-destination" that refer to the pair of end nodes wishing to exchange information. Assume that each source S_i has an infinite number of packets to transmit to its destination D_i. Denote by λ_i the long-term averaged number of packets per time slot that destination D_i successfully receives from source S_i. We are interested in examining how $\lambda = \min_i \lambda_i$ behaves asymptotically with n. In other words, what is the minimum throughput that we can guarantee

A routing/physical access strategy determines during each time slot a set of transmitters that attempt to transmit to a set of receivers. For example in [2], half of the nodes at each time slot attempt to transmit to their nearest neighbor, and the transmission is successful if the signal to noise and interference power is above a threshold. As a result, during each time slot, we get a configuration of simultaneously successful transmissions between sender-receiver pairs, which we call a *communication pattern*. Fig. 1 depicts possible such communication patterns during discrete time slots.

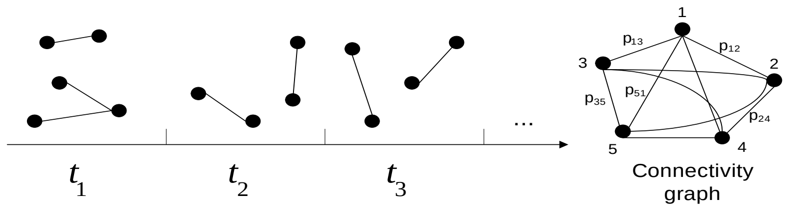

Figure 1: An adhoc network with $n = 5$ nodes. During each time slot a different communication pattern occurs.

We consider a specific pair of nodes i and j and calculate the probability p_{ij} that at a given time slot there exists a succeful comminucation connection between them. In other words, p_{ij} expresses the fraction of the time slots that nodes i and j are "connected", can directly and succesfully transmit to each other. Thus, p_{ij} upper bounds the fraction of packets per time slot that nodes i and j can exchange.

We construct a *connectivity graph* that connects every two nodes i, j with an edge $e = (i, j)$ of capacity $p(e) = p_{ij}$. The connectivity graph *does not* represent the actual ad-hoc network at any given time slot, but rather summarizes the probabilistic communication capabilities of the network. For example, this graph does not express either the correlation between successive instantiations of the graph, or the correlation among successful transmissions at any given time slot, but it still offers an abstraction of the communication capabilities of the original ad-hoc network, that we can use to study long-term averaged throughput. Indeed, routing in an ad-hoc network, corresponds to routing along paths in the connectivity graph.

For example, the connectivity graph associated with the model in [2] is a complete graph, where each edge (i, j) has capacity $p_{ij} = \Theta(1/n)$. Indeed, at a given time slot, the nodes i and j will be close to each other and will successfully exchange one unit of information with probability $\Theta(1/n)$. The connectivity graph captures this fact, that node i will be able to successfully send one information unit to every other node j, on the average once every n time slots. Thus, node i will be able to directly transmit to node j throughput $\Theta(1/n)$ per time slot.

To define the connectivity graph more formally, we can assume that the ad-hoc network can be modeled as a discrete time Markov process. Each state of the Markov process corresponds to a communication pattern, i.e., a set of point-to-point simultaneous successful transmissions during a time slot. For example in Fig. 1, during each time slot one such communication pattern - state of the Markov chain - occurs. This Markov process is a high-level/end-result model that incorporates the mobility model, the physical model of the network, the criterion for successful transmission between two nodes, and more generally depends on the factors that affect successful communication. Consider the steady-state probabilities associated with each state of the Markov process. For every two nodes i and j we compute the 'steady-state probability" p_{ij} that they can all states of the Markov process where node i successfully transmits to node j, i.e. all Markov states where i and j are connected. This in a way is a formal definition of how the probability p_{ij} can be calculated: in practice, we can directly calculate it using knowledge of the mobility pattern and the underlying physical model of the ad-hoc network, as we will demosntrate for a number of examples in Section 4.

If we consider "one-to-one" transmissions and not multicasting/broadcasting, the wireless medium implies that a node at each time slot can succesfully transmit to at most one receiver. As a result, a source can transmit throughput at most $\Theta(1)$ towards its destination, and consequently the destination cannot possibly receive more, i.e. $\lambda \leq \lambda_i \leq \Theta(1)$.

3 Conditions for $\lambda = \Theta(1)$

In this section, we are going to study whether constant throughput is possible on a given ad-hoc network, by examining necessary and sufficient conditions on the associated connectivity graph.

Consider a source-destination pair (S_i, D_i). To achieve throughput $\lambda_i = \Theta(1)$, source S_i needs to transmit information $\Theta(1)$ towards destination D_i. Accordingly, in the connectivity graph, there need to exist a set of paths through which source S_i will route information $\Theta(1)$ towards the destination D_i.

We are interested in the long-term average throughput that destination D_i will experience. If the source S_i would use the network by itself, and given that there exist paths of total capacity $\Theta(1)$, destination D_i would also receive throughput $\Theta(1)$. However, the network resources may need to be shared by more than one source-destination pair. Sharing resources means that, in the connectivity graph, a given edge may need to be used by more than one path. Thus, the throughput of that edge, is "divided" among the paths that share it. For example in the connectivity graph in Fig. 1, if two paths want to equally share an edge of capacity $\Theta(\frac{1}{n})$, then each path can at most route through that edge throughput $\Theta(\frac{1}{2n})$. Note that if a path intersects[2] with a finite number of other paths, the "order" of the throughput it conveys is not affected. We will show that if for each source-destination pair, there exists a set of paths of total capacity $\Theta(1)$ such that each path intersects with other paths a finite number of times, then it is possible to achieve $\lambda = \Theta(1)$. Conversely, we will show that if throughput $\lambda = \Theta(1)$ is possible, then there exist paths on the connectivity graph that convey this information from the source to the destination, and they intersect with other paths at most a finite number of times.

Theorem 1 *Consider the connectivity graph associated with a given ad-hoc network, and a set of randomly selected source-destination pairs (S_i, D_i). The following conditions are* **necessary and sufficient** *to achieve $\lambda = \Theta(1)$ throughput.*
For each (S_i, D_i) there exists a set of m_i paths $P^i = \{P_1^i, \cdots, P_{m_i}^i\}$ on the connectivity graph such that

1. *Source S_i transmits to D_i $\Theta(1)$ information units per time slot along the paths P^i.*
2. *Each path P_j^i intersects with other paths in a finite number of edges, where with intersection between two paths we refer to sharing common capacity of an edge.*

[2]By intersection we mean sharing common capacity of an edge in the connectivity graph.

1. *Sufficiency:* From construction and condition 1 there exists a set of paths $P^i = \{P_1^i, \cdots, P_{m_i}^i\}$ through which source S_i transmits $\Theta(1)$ packets per time slot towards destination D_i. Let $c(P_j^i)$ denote the number of packets per time slot routed from source S_i along the j-th path, then $c(P^i) = \sum_j c(P_j^i) = \Theta(1)$. Let λ_j^i, $j = 1 \ldots m_i$ denote the fraction of packets that D_i receives through path P_j^i, $\lambda_i = \sum_j \lambda_j^i$. We want to show that $\lambda_i = \Theta(1)$. If source S_i would use the network by itself, destination D_i would receive $\lambda_j^i = c(P_j^i)$. Since from condition 2 a finite number of edges in P_j^i are shared with other source-destination pairs, the order of throughput along the path is not affected, that is, $\lambda_j^i = \Theta(c(P_j^i))$. Thus $\lambda_i = \Theta(c(P^i)) = \Theta(1)$.
2. *Necessity:* Throughput $\lambda = \Theta(1)$ implies that for every (S_i, D_i) there exist on the connectivity graph a set of paths $P^i = \{P_1^i, \cdots, P_{m_i}^i\}$ such that $\sum_j c(P_j^i) = \Theta(1) = \sum_j \lambda_j^i$. Let $\sum_j \lambda_j^i = \sum_{P_j^i \in I} \lambda_j^i + \sum_{P_j^i \notin I} \lambda_j^i$, where I is the set of paths that intersect a finite number of times with other paths. If $\sum_{P_j^i \in I} \lambda_j^i = \Theta(1)$, we can ignore the paths that do not belong in I, and we are done.

Otherwise, $\sum_{P_j^i \notin I} \lambda_j^i = \Theta(1)$. Let path $P_j^i \notin I$ intersect $f_{ij}(n)$ times with other paths. Then there exists $g(n)$ such that $g(n) = O(f_{ij}(n))$ for each i, j and $\lim_{n\to\infty} g(n) = \infty$. But

$$\Theta(1) = \sum_{P_j^i \notin I} \lambda_j^i \leq \sum_{P_j^i \notin I} c(P_j^i)/g(n) = \Theta\left(\frac{1}{g(n)} \sum_{P_j^i \notin I} c(P_j^i)\right) = \Theta\left(\frac{1}{g(n)}\right),$$

which is a contradiction. □

The following corollary states that a sufficient condition to achieve constant throughput, is to find finite length information flow paths such that each edge of the connectivity graph is used a finite number of times.

Corollary 1 *The following conditions are* **sufficient** *to achieve* $\lambda = \Theta(1)$.
For each pair (S_i, D_i), *there exists a set of paths* $P^i = \{P_1^i, \cdots, P_{m_i}^i\}$ *such that*

1. *Source* S_i *transmits* $\Theta(1)$ *information units (packets) per time slot.*
2. *Each path* P_j^i *has finite length.*
3. *Each edge in the connectivity graph is used a finite number of times by the union of all paths between all source-destination pairs. That is,*

$$\sum_{i,j:\, e \in P_j^i} \lambda_j^i(e) = \Theta(p(e)), \tag{1}$$

where $p(e)$ *denotes the capacity of edge* e *and* $\lambda_j^i(e)$ *denotes the throughput along the path* P_j^i *through the edge* e.

Proof: Since from the second condition each path has finite length, the only way the second condition in Theorem 1 does not hold is if an edge is used an infinite number of times. But from condition 3 this is not possible. □

It is also easy, by using the connectivity graph properties, to find necessary conditions for throughput λ, where λ is not necessarily constant. For example, from the max-flow min-cut theorem, the min-cut in the connectivity graph between any k sources that transmit throughput λ to k destinations, has to be of order $k\lambda$.

one unit of information between each source-destination pair. For example if we use a path of length two to route throughput say c, we need to use $2c$ of the network capacity resources to deliver this information from the source to the destination. Generally,

$$\kappa_i = \sum_{j=1}^{m} l(P_j^i)\lambda_j^i, \tag{2}$$

where λ_j^i is the throughput routed through path P_j^i and $l(P_j^i)$ the number of edges of P_j^i. Then in total, all source-destination pairs, will need resources $\lambda \sum_i \kappa_i$ per unit time to deliver throughput λ. These required resources cannot exceed the available network resources $\kappa = \sum_e p(e)$, where $p(e)$ is the capacity of edge e. This intuitively explains the following proposition which we state without formal proof.

Proposition 1 *The following conditions are* **necessary** *to achieve throughput λ. For each pair (S_i, D_i), there exists a set of m_i paths $P^i = \{P_1^i, \cdots, P_{m_i}^i\}$ such that:*

1. *For any $k \geq 1$ source-destination pairs, the value of the min-cut between the k sources and the k destinations is greater or equal to $\lambda\Theta(k)$.*

2. *Let κ_i denote the total resources required by pair (S_i, D_i) calculated as*

$$\kappa_i = \sum_{j=1}^{m_i} l(P_j^i)\lambda_j^i, \tag{3}$$

where λ_j^i is the throughput routed along path P_j^i and $l(P_j^i)$ the number of edges of P_j^i. Let $\kappa = \sum_e p(e)$ be the total resources available at the network. Then

$$\lambda \sum_i \kappa_i = O(\kappa). \tag{4}$$

4 Applications

In this section we are going to apply the conditions in a number of examples, starting with the three cases in the literature we have discussed.

Example 1

In the Gupta-Kumar model [1] the nodes are static and transmit to nearest neighbors. The associated connectivity graph has constant degree c, and the weight associated with each edge is at most $1/c$. For each (S_i, D_i) pair there exist c max-flow paths, each carrying throughput $1/c$, of average length $\Theta(\sqrt{n})$. Thus, $\kappa_i = \Theta(\sqrt{n}c/c)$. From [1], $\kappa = \Theta(n)$, so from Eq. (4) we get

$$\lambda\Theta(n\sqrt{n}) = O(n) => \lambda = O(\frac{1}{\sqrt{n}}). \tag{5}$$

In the Grosglausser and Tse model [2], uniform mobility implies that the connectivity graph is a complete graph with uniform weight associated with every edge. That is, each node has degree $n-1$, and the weight associated with each edge is $\Theta(1/n)$ (corresponding to the probability that two nodes are nearest neighbours and the probability that a feasible sender receiver pair is scheduled).

We are going to apply the sufficient conditions in Corollary 1.

- From each source to each destination node, there exist $n-2$ edge-disjoint paths of length two, and one path of length one. Indeed, since the graph is complete, there exists an edge from node S_i to all other nodes in the graph (including the destination D_i) and an edge from D_i to every other node in the graph, which put together form the described max-flow paths. Using these paths, we can route throughput $\Theta(1) = (n-1)\frac{1}{n}$.

- We are going to show that if $\Theta(n)$ source-destination pairs share the network and use the paths previously described, each edge is used a finite number of times. Consider edge (k, l) between nodes k and l. This edge is going to be used only if node k is a source, or if node l is a destination - so at most two times.

Example 3

In the Diggavi, Grossglauser and Tse model [3] mobility is restricted since the nodes are only allowed to move along great circles in the surface of a sphere. However, we still obtain a similar connectivity graph to the [2] model, that is, the average degree for every node is of the order $\Theta(n)$ and the weights associated with each edge are $\Theta(1/n)$. Thus the analysis in the previous example still applies.

Example 4

Consider the rectangular grid depicted in Fig. 2 that has $2d$ lines. Assume that the nodes are uniformly distributed on the lines of the grid. Thus, each line will contain $n/2d$ nodes. Assume that the geometry of the problem is such that, nodes in the same line or in intersecting line can communicate with each other if the mobility/interference pattern makes it possible, but nodes in parallel lines cannot communicate. From Proposition 1, since the min-cut between parallel lines is at most equal to d, a necessary condition to have constant throughput is that $n/2d = O(d)$.

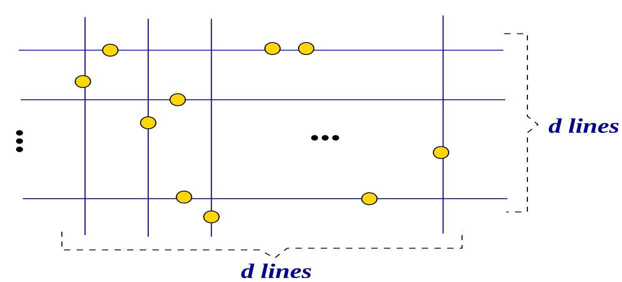

Figure 2: Mobility on a rectangular grid.

Now consider the case where $n = 2d$, that is, each line contains exactly one node. Then the corresponding connectivity graph is a complete bipartite graph, since from complete bipartite graph, if the source-destination pairs belong to parallel lines, there exist $n/2$ non-intersecting paths of length two, while if they belong to intersecting lines, there exist $n/2 - 1$ non-intersecting paths of length three and one path of length one. Thus, from Corollary 1, if each edge has capacity $\Theta(1/n)$, it is possible to achieve constant throughput.

If nodes are allowed to uniformly randomly move on their line, and ignoring edge-effects, the capacity of each edge is upper bounded by $\Theta(1/n^2)$. However, it is possible to construct a mobility pattern that leads to edge capacities $\Theta(1/n)$. For example, to avoid edge-effects we can assume that the nodes move on parallel (horizontal and vertical) "circles" instead of lines, ie. the "end points of the line segments are connected. We can then construct a mobility pattern, where nodes move clockwise on their circle, where at time slots $(i) mod(d) = k$, the node at the horizontal circle i meets with the node at the vertical circle $(i + k) mod(d)$.

5 Degree of Connectivity Graph

In the previous sections we have proposed a set of conditions to test whether $\Theta(1)$ throughput is achievable, but we have not discussed how these conditions translate in properties of the mobility pattern. To understand the implications for the mobility pattern, we can start by examining whether there exist fundamental structural properties of the connectivity graph that are necessary in order to achieve constant throughput.

This section focuses on possible restrictions that the degree of the connectivity graph should satisfy. The degree of the connectivity graph expresses the number of neighbors a given node may communicate with. We are going to assume that the degree is of the same order for all nodes, that is, all nodes behave in a similar way in terms of mobility.

As we saw in Example 1, if the connectivity graph has a constant degree, that is, every node can communicate with a fixed finite number of neighbors, it is not possible to achieve $\Theta(1)$ throughput. Indeed, as the number of nodes n increases, since the degree is constant, the average path length between a randomly chosen source-destination pair will also increase as a function of n, say $f(n)$. Moreover, in a wireless ad-hoc network with n nodes, we can have at most n successful connections per time slot, and thus $\kappa = O(n)$. But then from Eq. (4), $\lambda = O(\frac{1}{f(n)})$.

On the other hand, in both [2, 3], where $\Theta(1)$ throughput was possible, the associated connectivity graphs has degree $\Theta(n)$. The question we are looking at is, do the conditions on the connectivity graph imply that, to achieve throughput $\lambda = \Theta(1)$, it is necessary to have degree of order $\Theta(n)$? In other words, to achieve throughput $\lambda = \Theta(1)$ in an ad-hoc network with n nodes, is it a necessary condition that the mobility pattern ensures for every node to at some point be able to successfully communicate with $\Theta(n)$ other nodes?

Theorem 2 shows that the connectivity graph does not impose any such condition. There do exist possible connectivity graphs with degree of order $\Theta(n^{\frac{1}{t}})$, for finite t, such that the throughput is constant. Such connectivity graphs can be constructed from *de Bruijn* and *Kautz* graphs and their generalizations [4]. The *de Bruijn* graphs $D_B(d, t)$ have $n = d^t$ nodes, and degree at most $2d$. In the proof of theorem 2 we show that for each source-destination path there exist max-flow paths of length at most $2t$. For example, if we choose $d = \sqrt{n}$, $t = 2$, and associate weight $\Theta(1/\sqrt{n})$ with each edge, we have a graph that has degree $d = \Theta(\sqrt{n})$, and $\sqrt{n} - 1$ paths that carry flow $1/\sqrt{n}$ of length at most 4. Thus the conditions of Corollary 1 are satisfied.

that each edge has capacity $\Theta(1/d)$. Let $d = n^{\frac{1}{t}}$ and t finite. For any $\Theta(n)$ randomly chosen source-destination pairs, there exists max-flow paths of length at most $2t$ such that $\lambda = \Theta(\frac{1}{2t})$ and each edge of the graph is used a finite number of times.

Proof: Consider the de Bruijn graph $D_B(d,t)$ with $n = d^t$ nodes, and associate with each edge capacity $\Theta(1/d)$. We can think of every vertex of the graph as a t-dimensional vector over an alphabet of size d. Each node $(s_1\ s_2 \ldots s_t)$ connects to the d nodes $(s_2 \ldots s_t\ x)$, for every possible x in the alphabet.

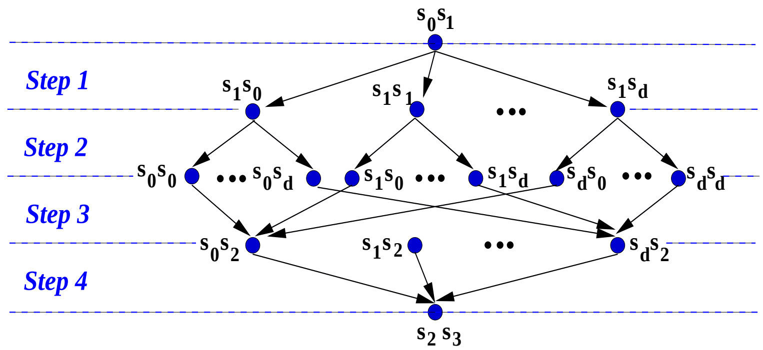

Figure 3: Proposed set of paths in the de Bruijn graph $D_B(\sqrt{n}, 2)$.

For simplicity we start by the case $d = \sqrt{n}$ and $t = 2$. Consider the source node $(s_0\ s_1)$ transmitting to a destination node $(s_2\ s_3)$. To route information we use the following length-four paths, that are also depicted in Fig. 3.

1. Source $(s_0\ s_1)$ transmits information $\Theta(1/\sqrt{n})$ to each of the $\sqrt{n}$ neighbors $\{(s_1\ s_0), (s_1\ s_1) \ldots (s_1\ s_d)\}$.

2. Each of the $\sqrt{n}$ nodes $(s_1\ s_i)$ equally distributes the $\Theta(1/\sqrt{n})$ information it has among its $\sqrt{n}$ neighbors $(s_i\ s_j)$. Thus at the end of this step, every node in the graph has $\Theta(1/n)$ of the information.

3. All nodes $(s_i\ s_j)$ transmit the $\Theta(1/n)$ information they have to the $\sqrt{n}$ nodes $(s_j\ s_2)$. Thus each $(s_j\ s_2)$ node collects $\Theta(1/\sqrt{n})$ of the information.

4. Finally, each $(s_j\ s_2)$ node transmits its information to the destination $(s_2\ s_3)$.

We show now that, if all $\Theta(n)$ source-destination pairs use the previously described paths, then each edge of the graph is employed a finite number of times. Let (ab, bc) be an arbitrary edge, we are going to calculate an upper bound on the number of times it is used, by bounding the number of times it may be employed through steps $1-4$.

1. Edge (ab, bc) is used at step 1 only if node ab is a source, and thus at most once.

2. Edge (ab, bc) is used at step 2 if there exists node xa that is a source. In that case it will carry information $\Theta(1/n)$ for the source xa. There exist at most $\sqrt{n}$ possible sources xa, and the edge (ab, bc) has capacity $1/\sqrt{n}$, thus to accommodate all of them (ab, bc) needs to be used at most once.

3. Edge (ab, bc) is used at step 3 only if there exists a node cx that is a destination. In that case it will carry information $\Theta(1/n)$ towards the destination cx. But there exist at most $\sqrt{n}$ possible destinations cx and thus to accommodate all of them the edge (ab, bc) needs to be used at most once.

it needs again to be used once.

Thus in total, each edge needs to be used at most $2t = 4$ times.

Generally, it is easy to see that, if we have degree $d = n^{\frac{1}{t}}$, and each edge has capacity $\Theta(\frac{1}{n^{\frac{1}{t}}})$ by following the same approach of equally distributing the information to all neighbors in t steps, until all nodes will have $\Theta(1/n)$ information for each source-destination pair, and then collecting the information towards the receiver in another t steps, we can construct max-flow paths of length $2t$ such that, each edge at every one of the $2t$ steps is used at most once. Indeed, at step k, $k = 1 \dots t$, each edge will carry for each source-destination pair load at most $\frac{1}{n^{\frac{(k+1)}{t}}}$ and will be used by at most $n^{\frac{k}{t}}$ pairs. Similarly for $k = t + 1 \dots 2t$. Note that the necessary condition in Eq. (4) is satisfied: the graph has a total of $nn^{\frac{1}{t}}$ edges, and the max-flow paths require $\Theta(n2tn^{\frac{1}{t}})$ edges. $\square$
From Theorem 2 and the previous discussion, we can see that

Corollary 2 *It is not possible to achieve constant throughput if the connectivity graph has constant degree. However, we may be able to achieve constant throughput if the connectivity graph has degree $n^{\frac{1}{t}}$, where t is a constant.*

Whether there exists a physically realizable mobility pattern that gives rise to a connectivity graph of degree $n^{1/t}$ is part of current research.

6 Conclusions

In this paper we proposed of necessary and sufficient conditions to achieve constant throughput in an ad-hoc network, and an initial attempt to understand what these conditions imply for the underlying mobility pattern. The conditions are based on the the connectivity graph which offers an abstraction of the communication capabilities of the ad-hoc network.

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